Prabhu Jagatbandhu College
Department of Mathematics

(Assignment-1,Maths(Hons) 1st year,2017)

(Pair of straight lines-I)

1. The vertices of a triangle lie on the st. lines $y = x\tan\theta_1$, $y = x\tan\theta_2$, $y = x\tan\theta_3$, the circumcentre being at the origin; prove that the locus of the orthocentre is the st. line
$x(\sin\theta_1 + \sin\theta_2 + \sin\theta_3) = y(\cos\theta_1 + \cos\theta_2 + \cos\theta_3)$
2. Show that the $eq^{\underline{n}}$ of the st. line joining the feet of the perpendiculars from $(d,0)$ on the st. lines $ax^2 + 2hxy + by^2 = 0$ is
$(a-b)x + 2hy + bd = 0$
3. If the st lines $ax^2 + 2hxy + by^2 = 0$ be two sides of a parallelogram and the st. line $lx + my = 1$ be one of its diagonals then show that the $eq^{\underline{n}}$ of the other diagonal is $y(bl - hm) = x(am - hl)$
4. Prove that the orthocentre (α, β) of the triangle formed by the st. lines $ax^2 + 2hxy + by^2 = 0$ and $lx + my = 1$ is given by
$$\frac{\alpha}{l} = \frac{\beta}{m} = \frac{a+b}{am^2 - 2hlm + bl^2}$$
5. Show that the distance from the origin to the orthocentre of the triangle formed by the st lines $\frac{x}{\alpha} + \frac{y}{\beta} = 1$ and $ax^2 + 2hxy + by^2 = 0$ is
$$\frac{\alpha\beta(a+b)(\alpha^2 + \beta^2)^{\frac{1}{2}}}{a\alpha^2 - 2h\alpha\beta + b\beta^2}.$$

Date:22/08/17

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(Assignment-II,Maths(Hons) 1st year,2017)

(Pair of straight lines-II)

1. The st. line $ax+by+c=0$ bisects an angle between a pair of st. lines of which one is $lx+my+n=0$. Show that the other line of the pair is $(lx+my+n)(a^2+b^2)-2(al+bm)(ax+by+c)=0$
2. If the $eq^{\underline{n}}$ $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a pair of st lines, then show that the area of the triangle formed by the bisectors of the angles between them and the axis of x is $\frac{\sqrt{(a-b)^2+4h^2}}{2h}\cdot\frac{ca-g^2}{ab-h^2}$ or $\frac{\sqrt{(a-b)^2+4h^2}}{2h}\left(\frac{gh-af}{ab-h^2}\right)^2$ (show both results)
3. If the $eq^{\underline{n}}$ $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a pair of intersection st. lines then show that the square of the distance of the point of intersection of the st. lines from the origin is $\frac{c(a+b)-f^2-g^2}{ab-h^2}$
4. Show that the area of the parallelogram formed by $ax^2+2hxy+by^2+2gx+2fy+c=0$ and $ax^2+2hxy+by^2-2gx-2fy+c=0$ is $\frac{2c}{\sqrt{h^2-ab}}$
5. If the $eq^{\underline{n}}$ $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents two parallel st. lines then show that the distance from them is $2\sqrt{\frac{g^2-ac}{a(a+b)}}$ or $2\sqrt{\frac{f^2-bc}{b(a+b)}}$ (show both results).
6. The st. lines joining the origin to the common points of the curve $ax^2+2hxy+by^2=c$ and the st. line $lx+my=1$ are at right angles. Show that the locus of the foot of the perpendicular from the origin on the st. line is $(a+b)(x^2+y^2)=c$

Date:12/9/17

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(Assignment-III,Maths(Hons) 1st year,2017)

(Canonical Form)

1. Discuss the nature of the conics and find centre and eccentricity
   $3x^2 - 2xy + 3y^2 - 4x - 4y - 12 = 0 \quad and \quad x^2 - 6xy + y^2 - 4x - 4y + 12 = 0$
2. Discuss the nature of the conics and find axis, equation of latus rectum, focus(if possible) $4x^2 - 4xy + y^2 - 8x - 6y + 5 = 0$, $x^2 + 4xy + 4y^2 + 4x + y - 15 = 0$
3. Reducing the equation $4x^2 + 4xy + y^2 - 4x - 2y + a = 0$ to its canonical form, determine the nature of the conic for different values of a.
4. Discuss the nature of the conic $x^2 + 2xy + y^2 - 4x - 4y + 3 = 0$.

Date:25/10/17

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(Assignment-IV,Maths(Hons) 1st year,2017)

(Tangents)

1. Tangents are drawn to the parabola $y^2 = 4ax$ at the points whose abscissa are in the ratio $p:1$. Show that the locus of their point of intersection is a parabola.
2. Show that the locus of the point of intersection of tangents to the parabola $y^2 = 4ax$ at points whose ordinates are in the rat $p^2 : q^2$ is $y^2 = \left(\frac{p^2}{q^2} + \frac{q^2}{p^2} + 2\right)ax.$
3. Tangents are drawn from (x_1, y_1) to the circle $x^2 + y^2 = a^2$; prove that the area of the triangle formed by them and the straight line of joining their contact is
$$\frac{a(x_1^2 + y_1^2 - a^2)^{\frac{3}{2}}}{x_1^2 + y_1^2}$$
4. Find the area the triangle formed by the tangents from the point (h, k) to the parabola $y^2 = 4ax$ and the chord of contact.
5. An ellipse is rotated through a right angle in its own plane about its centre, which is fixed. Prove that the locus of the point of intersection of a tangent to the ellipse in its original position with the tangent at the same point of the curve in the new position is $(x^2 + y^2)(x^2 + y^2 - a^2 - b^2) = 2(a^2 - b^2)xy.$

Date:

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(Straight lines)

Answer all questions:

1. Show that the equations of the projection of the straight line $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z-3}{4}$ on the plane x+2y+z=6 are $\frac{x-3}{4}=\frac{y+2}{-7}=\frac{z-7}{10}$
2. Show that the equation of the plane through the line $\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$ and perpendicular to the plane containing the lines $\frac{x}{m}=\frac{y}{n}=\frac{z}{l}$ and $\frac{x}{n}=\frac{y}{l}=\frac{z}{m}$ is (m-n)x+(n-l)y+(l-m)z=0
3. Show that the equation of the plane containing the straight line $\frac{y}{b}+\frac{z}{c}=1, x=0$ and parallel to the straight line $\frac{x}{a}-\frac{z}{c}=1, y=0$ is $\frac{x}{a}-\frac{y}{b}-\frac{z}{c}=1$ and 2d be the SD between the lines, then show that $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{d^2}$
4. A straight line is parallel to the plane y+z=0 and the intersects the circles $x^2+y^2=a^2, z=0$ and $x^2+z^2=a^2, y=0$ show that it generates the surface $x^2+(y+z)^2=a^2$
5. Show that the locus of the straight line which moves parallel to the xz plane and meets the curve $xy=c^2, z=0; y^2=4cz, x=0$ is $(c^2-xy)(y^2-4cz)=4cxyz$
6. A variable line intersects the lines y=0,z=c ; x=0, $z=-c$ and is parallel to the plane lx+my+nz=p. show that the surface generated by it is $lx(z-c)+my(z+c)+n(z^2-c^2)=0$

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