Prabhu Jagatbandhu College
Department of Mathematics
Assignment-4
(Soft copy is available in www.sites.google.com/site/pjbcmath15/)

1. State and prove Intermediate Value Theorem.

2. i) If f is a continuous function on [a, b] then show that |f(x)| is also continuous. Is the converse true? Justify your answer.

ii) If $f:[0,1] \to R$ is a function defined by $f(x)=\begin{cases} 1 & x \in Q \\ 0 & x \notin Q \end{cases}$, show f is totally discontinuous on [0,1] .

3. Prove that a bounded sequence $\{x_n\}$ of real numbers converges iff $\overline{\lim}(x_n)=\underline{\lim}(x_n)$

4. State Archimedean property and write down the geometrical significance of it and deduce the following two statements:

a) if $x \in R$ and $x > 0$ then there exists a natural number n such that $0<\frac{1}{n}<x$.
b) if $x\in R$, there exists an integer m such that $m-1 \le x < m$.

5. a) Prove that the intersection of finite number of open sets is open.
b) Is the above result is true for infinite number of open sets? Justify your answer.

6. Let S be a bounded subset of R with supS = M and infS= m. Prove that the set $T = \{x-y : x\in S, y\in S\}$ is a bounded set and supT = M-m and inf T= m-M.

7. Define Lipschitz function on an interval $I \subseteq R$. Give an example of a Lipschitz function defined on I. Let $f: I \to R$ be a Lipschitz function on I, then prove that f is uniformly continuous on I.

8. Find the general solution in positive integers of the equation $12x - 7y = 8$.

9. If the sum of the roots of the equation $x^4 + mx^2 + nx + p = 0$ is equal to the product of other two roots then examine whether $(2p - n)^2 = (p - n)(p + m - n)^2$.

10. Discuss the convergence of the sequence $\{S_n\}$, where $S_{n+1}=\sqrt{\frac{ab^2+S_n^2}{a+1}}$,b>a for all n>1 and $S_1 = a > 0.$

11. Show that the locus of the poles of tangents to the parabola $y^2=4ax$ with respect to the parabola $y^2=4bx$ is the parabola $y^2=\frac{4b^2}{a}x$.

12. (i) If $I_{m,n} = \int_0^{\pi/2} \sin^m x \cos^n x dx$, show that $I_{m,n}=\frac{m-1}{m+n} I_{m-2,n}$. Hence evaluate $\int_0^{\pi/2} \sin^6 x \cos^2 x dx$

(ii) Obtain the reduction formula for $\int \frac{dx}{(x^2+a^2)^n}$, where n being a positive integer greater than 1.

Submission Deadline: After Puja Vacation.